Math 111 **Annuity Solutions** **Spring 2018**

1. The values given are $P = 400$, $r = 4\% = 0.04$, $k = 12$, and $t = 5$. So $r/k = 0.04/12$ and $kt = 60$. Calculate F:

$$\begin{aligned} F &= P\left(\frac{\left(1+\frac{r}{k}\right)^{kt}-1}{r/k}\right) \\ &= 400\left(\frac{\left(1+\frac{0.04}{12}\right)^{60}-1}{0.04/12}\right) \\ &= 400(66.29897818) \\ &= \$26,519.59. \end{aligned}$$

2. His monthly pay is \$4,000 and 5% of that is \$200.00, so $P = 200$. Also, $r = 8\% = 0.08$, $k = 12$, and $t = 45$. So $r/k = 0.08/12$ and $kt = 540$. Calculate F:

$$\begin{aligned} F &= P\left(\frac{\left(1+\frac{r}{k}\right)^{kt}-1}{r/k}\right) \\ &= 200\left(\frac{\left(1+\frac{0.08}{12}\right)^{540}-1}{0.08/12}\right) \\ &= 200(5274.539892) \\ &= \$1,054,907.98. \end{aligned}$$

3. 10% of his income is \$400 a month, so replace 200 with 400 in the previous problem:

$$\begin{aligned} F &= 400(5274.539892) \\ &= \$2,109,815.96. \end{aligned}$$

4. Now we are using $P = 400$ and $r = 10\% = 0.10$, so we must compute F from the start:

$$\begin{aligned} F &= P\left(\frac{\left(1+\frac{r}{k}\right)^{kt}-1}{r/k}\right) \\ &= 400\left(\frac{\left(1+\frac{0.10}{12}\right)^{540}-1}{0.10/12}\right) \\ &= 400(10482.50171) \\ &= \$4,193,000.68. \end{aligned}$$

5. To find the future cost of tuition, rising at 6% per year, compute

$$(52000)(1.06)^{18} = \$148,425.64.$$

So he wants to save up \$148,425.64 over 18 years. We have $F = 148425.64$, $r = 0.09$, $k = 12$, and $t = 18$. Then $r/k = 0.09/12 = 0.0075$ and $kt = 216$. Calculate P:

$$\begin{aligned} F &= P\left(\frac{\left(1+\frac{r}{k}\right)^{kt}-1}{r/k}\right) \\ 148425.64 &= P\left(\frac{(1.0075)^{216}-1}{0.0075}\right) \\ &= P(536.351674). \end{aligned}$$

Divide to get

$$\begin{aligned} P &= \frac{148425.64}{536.351674} \\ &= \$276.73. \end{aligned}$$

6. We are given $P = 200000$, $r = 0.10$, $k = 2$, $t = 4$, so $r/k = 0.05$ and $kt = 8$. Calculate M:

$$\begin{aligned} M &= P\left(\frac{r/k}{1-\left(1+\frac{r}{k}\right)^{-kt}}\right) \\ &= 200000\left(\frac{0.05}{1-(1.05)^{-8}}\right) \\ &= 200000(0.1547218136) \\ &= \$30,944.36. \end{aligned}$$

7. In the formula, replace 200000 with 150000 and replace 0.05 with $0.06/2 = 0.03$. Calculate M:

$$\begin{aligned} M &= P\left(\frac{r/k}{1-\left(1+\frac{r}{k}\right)^{-kt}}\right) \\ &= 150000\left(\frac{0.03}{1-(1.03)^{-8}}\right) \\ &= 150000(0.1424563888) \\ &= \$21,368.46. \end{aligned}$$

8. We are given $P = 1000000$, $r = 0.08$, $t = 25$. Because $k = 1$ (annual withdrawals), we can use the simpler formula.

$$\begin{aligned} M &= P\left(\frac{r}{1-(1+r)^{-t}}\right) \\ &= 1000000\left(\frac{0.08}{1-(1.08)^{-25}}\right) \\ &= 1000000(0.0936787791) \\ &= \$93,678.78. \end{aligned}$$

9. We are given $M = 6000$, $t = 20$, $k = 12$, and $r = 0.09$. So $kt = 240$ and $r/k = 0.09/12 = 0.0075$. Write the equation, substitute the values, and solve for P:

$$\begin{aligned} M &= P\left(\frac{r/k}{1-\left(1+\frac{r}{k}\right)^{-kt}}\right) \\ 6000 &= P\left(\frac{0.0075}{1-(1.0075)^{-240}}\right) \\ 6000 &= P(0.0089972596). \end{aligned}$$

Now divide to get P:

$$\begin{aligned} P &= \frac{6000}{0.0089972596} \\ &= \$666,869.72. \end{aligned}$$

10. We have to work this problem in two parts. First, we must find out how much money he must save up in order to be able to withdraw \$5,000 each month for 20 years. That is like Problem 9, but with the values $M = 5000$, $t = 20$, $k = 12$, and $r = 0.09$ and then $kt = 240$ and $r/k = 0.0075$. (Note that only one number changed from Problem 9.) Now solve for P:

$$\begin{aligned} M &= P\left(\frac{r/k}{1-\left(1+\frac{r}{k}\right)^{-kt}}\right) \\ 5000 &= P\left(\frac{0.0075}{1-(1.0075)^{-240}}\right) \\ 5000 &= P(0.0089972596). \end{aligned}$$

Now divide to get P:

$$\begin{aligned} P &= \frac{5000}{0.0089972596} \\ &= \$555,724.77. \end{aligned}$$

For the second part, we must find out how much he should invest each month for 50 years in order to accumulate \$555,724.77. We must solve for P (payment), given $F = 555724.77$ (future value), $r = 0.09$, $t = 50$, $k = 12$, $r/k = 0.0075$, and $kt = 600$, using the formula for *building up* an annuity.

$$
\begin{aligned}
F &= P\left(\frac{\left(1+\frac{r}{k}\right)^{kt}-1}{r/k}\right)\\
555724.77 &= P\left(\frac{(1.0075)^{600}-1}{0.0075}\right)\\
&= P(11669.10186).
\end{aligned}
$$

Divide to get

$$
\begin{aligned}
P &= \frac{555724.77}{11669.10186}\\
&= \$47.62.
\end{aligned}
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